ON "BIMODALITY OF THE SOLAR CYCLE" AND THE DURATION OF CYCLE 21

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Abstract

The "period-growth dichotomy" of the solar cycle predicts that cycle 21, the present solar cycle, will be of long duration (> 133 months), ending after July 1987. Bimodality of the solar cycle (long- or short-period cycles) is clearly seen in a scatter diagram of descent versus ascent durations. Based on the well-observed cycles 8-20, a linear fit for long-period cycles (being a relatively strong inverse relationship, $r^2 > 0.66$, that is significant at the 5% level) suggests that cycle 21, having an ascent of 42 months, will have a descent near 99 months, yielding a cycle duration of about 141 months. Like cycle 11, cycle 21 occurs on the downward envelope of the sunspot number curve, yet is associated with an upward "first difference" in amplitude. A comparison of individual cycle, smoothed sunspot number curves for cycles 21 and 11 reveals striking similarity, which suggests that if, indeed, cycle 21 is a long-period cycle, then it too may have an extended tail of sustained, low, smoothed sunspot number, with cycle 22 minimum occurring either in late 1987 or early 1988.

1. Introduction

The duration of the solar cycle, the time between successive smoothed sunspot minima, based on the well-observed sunspot record which extends from cycle 8 to the present, averages about 131.6 months. However, as noted by Wilson (1984) and Sargent (1984a, b), solar cycles occur in "strings," lasting at least 2 consecutive cycles, being either of "short-period" or "long-period" duration (cf. Dicke, 1978, 1979; Yoshimura, 1979). Wilson (1984) has found short-period cycles to have a duration shorter in length than 128 months and long-period cycles to have a duration longer in length than 133 months; no solar cycle has a duration falling in the range of 128-133 months, a range which includes the mean of the entire distribution of cycle durations. (This is also true if one includes the cycle durations for the less well-determined cycles 1-7.) Wilson has further noted (cf. Waldmeier, 1935; Kiepenheuer, 1953; Bracewell, 1985) that short-period cycles often are cycles of higher-than-average smoothed sunspot number at cycle maximum R_M, while long-period cycles often are cycles of lower-than-average R_M. (For cycles 8-20, R_M averages about 116.)

In a subsequent study, Rabin, Wilson, and Moore (1986) have reported a <u>stronger</u> relationship in the solar cycle, one coupling cycle amplitude "trend" and cycle duration. They have found that the sunspot cycle is always shorter than the mean cycle duration when the upper envelope of the sunspot number curve is rising and always longer when it is falling. They have called this relationship the "period-growth dichotomy," and have suggested that the solar cycle has two distinct physical modes: "short-while-growing" and "long-while-declining." Based on the current apparent trend in the "period-growth dichotomy," they have predicted that the present cycle (21) will be longer than 133 months, ending after July 1987.

In this paper, additional comments are given concerning the "bimodality of the solar cycle" and the duration of cycle 21. In particular, the distribution of descent duration against ascent duration for the well-observed solar cycles is shown to be better described in terms of short-period and long-period cycle groupings, rather than in terms of a single collective grouping. Also, a comparison of smoothed sunspot number curves for cycles 21 and 11, both regarded to be "anomalous" cycles according to their "first difference" in amplitude, is given which reveals them to be strikingly similar.

2. Discussion

2.1 Comparison of Cycles 1-7 and 8-20

In the Rabin, Wilson, and Moore (1986) study, cycle durations and amplitudes were compared for cycles 1-20, spanning the years 1755-1976. In this investigation, however, one which makes use of ascent and descent durations, the timespan has been reduced to cycles 8-20 (1833-1976). Cycles 1-7 (1755-1833) usually are differentiated from cycles 8-20, in that the latter cycles are better determined in comparison to the former cycles (cf. McNish and Lincoln, 1949; Eddy, 1977; Wilson, 1984). This is readily seen from Waldmeier (1961), which contains a complete listing of daily sunspot number values for the interval 1849 through 1960, but only a partial listing prior to 1849, often being degraded (less data) for earlier dates extending back to 1818. Prior to 1818, Waldmeier gives no daily sunspot number values.

Table 1 summarizes the results of a comparison between cycles 1-7 and 8-20 for selected solar cycle parameters, including R_M, ascent duration ASC, descent duration DES, and cycle duration or period PER. The comparison reveals that, although corresponding mean values for these parameters may be insignificantly different at the 95% or 90% levels of confidence, corresponding <u>variances</u> sometime show large differences, with cycles 1-7 having the larger variances.

The ratios of variance between cycles 1-7 and 8-20 indicate that R_M and PER may be more reliably determined than DES or ASC (that is, the ratios for R_M and PER are closer to unity). Accepting values of means and standard deviations for cycles 8-20 to be equivalent to population values for the selected parameters shown in Table 1 and applying the zM Test (Langley, 1971), one finds the following: differences in mean values for R_M and PER for cycles 1-7 and 8-20 are insignificantly different and, thus, may be due entirely to chance; however, differences in mean values for ASC and DES are significantly different at $\lesssim 0.2\%$ and $\lesssim 5\%$, respectively. Therefore, for ASC and DES, the differences in mean values for cycles 1-7 and 8-20 appear to be genuine and not just a variation due to chance. Consequently, one argues that values of ASC and DES for cycles 1-7 should not be pooled together with those of cycles 8-20; they are inherently different, reflecting either a real difference in the early cycles as compared to modern solar cycles or, perhaps, a lack of reliability in the placement of R_M in the earlier cycles, due to poorly monitored sunspot observations. For this investigation, since the primary focus is on values of ASC and DES, only those values for the well-observed cycles 8-20 will be employed. The pooling of values of R_M and PER, however, as was done in the investigation of Rabin, Wilson, and Moore (1986), was both acceptable and desirable -- acceptable because the differences are very probably due to chance, and desirable because a larger data set became available with the inclusion of cycles 1-7.

2.2 Bimodality of the Solar Cycle

As previously noted, short-period cycles tend to be associated with cycles of higher-than-average smoothed sunspot number at cycle maximum R_M, while long-period cycles tend to be associated with cycles of lower-than-average R_M (cf. Wilson, 1984; Wilson, Reichmann, and Teuber, 1984; Bracewell, 1985). Based on cycles 8-20, 4 of 6 "high" cycles are of short-period cycle duration, while 5 of 7 "low" cycles are of long-period cycle duration. On the basis of a 2x2 classification, one can easily compute the probability P that no preferential association exists in the observed distribution of cycle amplitude class (high or low) and cycle mode (short or long) using Fisher's Exact Test (Langley, 1971; Everitt, 1977). The resultant probability is $P \approx 18\%$, and the probability of obtaining the observed result or one more suggestive of a departure from independence is $P \approx 21\%$. Consequently, one cannot confidently predict cycle duration on the basis of cycle amplitude class.

Since the ascent duration ASC of a cycle is directly related to its amplitude at cycle maximum R_M, in that cycles of shorter-than-average ASC are "high" R_M cycles (5 of 6, of cycles 8-21), while cycles of longer-than-average ASC are "low" R_M cycles (6 of 8), a result significant at the 95% confidence level ($P \approx 2.5\%$ for obtaining the observed result or one more suggestive of a departure from independence), a similar distribution between ascent duration class and cycle mode, as between cycle amplitude class and cycle mode, should exist; in fact, it does. Therefore, as concluded above for amplitude class, one cannot confidently predict cycle duration on the basis of cycle ascent duration class.

Instead of comparing cycle mode and amplitude class or ascent duration class, as above, for cycles 1-20, one compares cycle mode and the "first difference" in amplitude (up or down), where the "first difference" $\Delta(n)$ is the difference between R_M values for two successive cycles, one can determine, as before, the probability P that no preferential association exists in the observed distribution. The resultant probability is $P < 1\%$ ($P \approx 0.3\%$ for obtaining the observed result or one more suggestive of a departure from independence). Thus, an upward "first difference" in amplitude tends to associate with cycles of short-period duration, while a downward "first difference" tends to associate with cycles of long-period duration. While the association is significant, some notable exceptions have occurred. Namely, cycles 11 and 13, both having an upward "first difference," are long-period cycles, and cycle 16, having a downward "first difference," is a short-period cycle. A moderately strong ($r^2 \approx 0.26$) inverse relation between PER and Δ is found from regression analysis, approximated as $\hat{Y} = 131.9 - 0.1\ X$, where $\hat{Y}$ is the expected value of PER and X is the observed value of $\triangle$; the standard error of estimate S_{YX} is about 9.3.

If, instead of using the "first difference" in amplitude for cycles 1-20, one uses the actual, albeit somewhat subjective, bimodal "trend arrows" which ignore single cycle reversals (see Figure 1 in Rabin, Wilson, and Moore, 1986), then, one observes that <u>all</u> cycles of a downward trend are long-period cycles (11 of 11) and <u>all</u> cycles of an upward trend are short-period cycles (9 of 9). The probability P that no preferential association exists in the <u>observed</u> distribution is computed to be $P \ll 1\%$ ($P \approx 0.0006\%$). Cycle 21 has an upward "first difference" in amplitude, implying that it is a short-period cycle; however, as in the cases of cycles 11 and 13 which also had upward "first differences," yet were part of a general downward trend according to the bimodal trend arrow, cycle 21 should be a long-period cycle.

In Table 2, durations of ascent ASC, descent DES, and period PER are identified for cycles 8-20. Means, standard deviations, and 95% and 90% confidence intervals for the means also are given, as is the cycle mode for each cycle (S = short-period cycle; L = long-period cycle). Thus, an average solar cycle has an ascent duration of 48.2 ± 3.9 months, a descent duration of 83.5 ± 6.3 months, and a cycle duration of 131.6 ± 6.3 months, using 95% confidence intervals.

To ascertain the "goodness of fit" of the observed distribution of each of these parameters, as compared to a Gaussian or normal distribution, one can apply the Kolmogorov-Smirnov Test (Lapin, 1978). Based on values contained in Table 2 for cycles 8-20, one concludes that ASC, DES, and PER may be distributed normally at the 95% or 90% levels of confidence. The greatest deviation, although still at a level insignificantly different from the normal distribution at the stated levels of significance, is for PER. By invoking bimodality (long- and short-period distributions, each normally distributed about its respective mean), one can reduce the deviation in PER. Furthermore, its variance can be reduced from 108.2 to 39.0. While this is attractive, one must note that in doing so, the variance in ASC increases from 42.2 to 80.9, while it only slightly decreases for DES, from 108.2 to 103.9. (A Kolmogorov-Smirnov Test, comparing the observed distribution to a uniform distribution, likewise, results in the acceptance of the null hypothesis -- that is, ASC, DES, and PER may be distributed uniformly. Thus, there is no significant evidence to reject either the normal or uniform distributions for ASC, DES, or PER. A larger data set is needed.)

Another valuable statistical test that can be readily applied to the data is the Wilcoxon's Sum of Ranks Test (Langley, 1971). This test allows one to evaluate the effect of "bimodality of the solar cycle" in the distributions of ASC and DES. Application of the test results in the rejection of the null hypothesis (difference in sum of ranks due to chance) for DES at the 5% level of significance; the null is accepted at the 1% significance level. For ASC, the null hypothesis is accepted even at the 90% level of confidence. Thus, for DES, one can be assured at $\gtrsim$ 95% level of confidence that a bimodal distribution exists, and that the difference in sum of ranks cannot be due to chance; for ASC, one must accept at the 90% level of confidence that its sum of ranks is due to chance.

One additional, possibly useful comparison is to contrast "expected" and "observed" values for PER, assuming a normal distribution and using five standardized sorting bins, each bin being one standard deviation wide, with the central bin centered on the mean of the distribution. Doing so, one obtains the following "observed" sequence of bin frequency for PER: 1-5-2-4-1. For a Gaussian or normal distribution of 13 solar cycles, the "expected" sequence is approximately 0.8-3.1-5.0-3.1-0.8. Thus, for PER, a possibly meaningful discrepancy is discerned. (One should note that the "observed" sequence of 1-5-2-4-1 is approximately the same as the "expected" sequence of 0.4-5.0-1.8-4.5-0.4, which results when one combines the two "expected" sequences of the normal distributions for short-period and long-period cycles, using the bins of the unimodal distibution.)

Based on a 2x2 classification, comparing observed and expected values for PER in the central bins (containing the means) with all remaining bins, one can determine, as before, the probability P that no preferential association exists; this value is computed to be $P \approx 15\%$. Also, the probability of getting the observed result can be directly computed from the binomial formula, for $n = 13$ (the number of solar cycles) and $r = 11$ (the number of cycles with values of PER outside the central bin). Its value is computed to be $P \approx 6\%$. (In the calculation $\pi \approx 8/13$ for the "expected" distribution.) A Runs Test (Stevens, 1939; Wald and Wolfowitz, 1940; and Swed and Eisenhart, 1943) for PER, to determine the probability that the sequence of alternatives is random, shows that, for cycles 8-20, the sequence of alternatives, consisting of 6 short-period cycles and 7 long-period cycles in 4 runs, has a probability for randomness P $\approx$ 9%. Based on the larger data set of cycles 1-20, consisting of 9 short-period cycles and 11 long-period cycles in 7 runs, one computes P $\approx$ 4%. Thus, confidence that a bimodal distribution of PER exists certainly at the 90% level and probably even at the 95% level.

In Table 3, means, standard deviations, and 95% and 90% confidence intervals for the means are tabulated separately for the two presumed cycle groupings: short-period cycles (S) and long-period cycles (L). A short-period cycle, on average, has an ascent duration of 45.8 $\pm$ 7.0 months, a descent duration of 76.0 $\pm$ 7.3 months, and a cycle duration of 121.8 $\pm$ 3.8 months, using the 95% confidence intervals. In contrast, a long-period cycle, on average, has an ascent duration of 50.1 $\pm$ 5.6 months, a descent duration of 89.9 $\pm$ 7.9 months, and a cycle duration of 140.0 $\pm$ 4.7 months, using 95% confidence intervals.

In Figure 1a, the individual cycle lengths for cycles 8-20 are plotted, and the mean values for the two presumed cycle duration groupings and for the combined group, ignoring bimodal classification, are given. In Figure 1b, values of descent DES and ascent ASC for the same solar cycles are depicted, as are the mean values of ascent <ASC> and descent <DES>. For clarity, the individual points in Figures 1a and 1b have been plotted as filled circles (●)

and filled triangles (▲), distinguishing short-period and long-period cycles, respectively. Notice that 6 of 7 long-period cycles have descent durations longer than <DES>, while 6 of 6 short-period cycles have descent durations shorter than <DES>. Based on the binomial formula, one computes the probability of getting these observed distributions to be $P \approx 6\%$ and $P \approx 2\%$, respectively. Thus, a bimodal classification in DES versus ASC is quite apparent, at least by quadrant, with long-period cycles tending to populate quadrants I and II and with short-period cycles populating quadrants III and IV. Based on a 2x2 classification with respect to <DES> and <ASC>, one computes the probability P that no preferential association exists in the observed distribution, ignoring bimodality of the solar cycle, to be $P \approx 31\%$; the probability of obtaining the observed result or one more suggestive of a departure from independence is $P \approx 38\%$. Thus, it seems very unlikely that DES is strongly related to ASC for the overall distribution, when bimodal grouping is disallowed. A general linear fit (not shown) of DES to ASC confirms this; namely, the regression, approximated as $\hat{Y} = 107.9 - 0.5 X$, where $\hat{Y}$ is the expected value of DES and X is the observed value of ASC, has a correlation coefficient r of only -0.32, indicating, at best, that the correlation is weak. The standard error of estimate S_{yx} is about 10.3. (One should note that application of Spearman's Correlation Test -- Langley, 1971 -- shows that DES is not strongly correlated with ASC, even at the 10% significance level.)

By invoking bimodality of the solar cycle and applying a linear fit to each of the groups, one obtains fits which are seemingly more representative of the data. These linear fits are illustrated in Figure 1b by the two lines called "long-period cycles" and "short-period cycles." Long-period cycles are better described by the upper regression, approximated as $\hat{Y} = 147.2 - 1.1\ X$, where $\hat{Y}$ and X have the same meanings as before, which has $r = -0.81$ and $S_{yx} = 5.5$. Short-period cycles are better described by the lower regression, approximated as $\hat{Y} = 116.8 - 0.9\ X$, which has $r = -0.86$ and $S_{yx} = 4.2$. While the slopes are insignificantly different from each other at either the 95% or 90% confidence levels, they are both statistically different from the null slope (= 0) at the 95% level of confidence. Thus, the difference is seen to be largely a factor of the Y-axis intercept. In all fits, the computed slope is negative, indicating that DES is inversely correlated with ASC; i.e., cycles with short (long) ASC have long (short) DES. (Application of Spearman's Correlation Test reveals that the relatively strong inverse relations are significant at about the 5% level.)

2.3 Duration of Cycle 21

As stated before, the present cycle (21) is expected to be a long-period cycle, having a duration > 133 months. Applying the value of its ASC (= 42) to the linear fit for long-period cycles yields the value of its DES ($\approx$ 99), a value remarkably close to that observed for cycle 11 (= 100). The 95% confidence interval for DES for cycle 21 is DES = 99.2 ± 17.1 months. (If, on the other hand, cycle 21 is a short-period cycle, although this is considered highly unlikely in light of the "period-growth dichotomy," then, the 95% confidence interval for DES for cycle 21 is DES = 79.4 ± 14.5 months.) Thus, one can summarize as follows: (1) the "period-growth dichotomy" predicts that cycle 21 is a long-period cycle of duration > 133 months; (2) the mean duration of long-period cycles is about 140.0 ± 4.7 months (at the 95% confidence level), indicating that cycle 21 will have a cycle duration > 135 months; and (3) the linear fit for long-period cycles yields an expected value for cycle 21 DES near 99 months, suggesting that the cycle duration for cycle 21 will be about 141 months.

2.4 Comparison of the "Anomalous" Cycles 21 and 11

Cycle 21, like its predecessor and many other cycles, has had recurrent bursts of activity, which tend to lengthen the cycle. Further, it shares several common characteristics with cycle 11. For example, both cycles 21 and 11 occur on the downward trend of the envelope of R_M, are (or are anticipated to be) cycles of long duration, have upward "first differences" in amplitude, have R_M values higher than average, and have ASC values of comparable duration which are shorter than average. Thus, it may be that the known smoothed sunspot number curve for cycle 11 may yield insight into the behavior of cycle 21 for the remainder of its decline.

In Figure 2, the smoothed sunspot number curves for cycles 21 and 11 are compared. A remarkable similarity is readily seen. Both cycles, as already mentioned, have R_M values higher than average, with cycle 21 being the second highest amplitude cycle on record (based on cycles 8-21) and cycle 11 being the fifth highest. Furthermore, both display relatively smooth, rapid rises to single, well-defined peaks, and decay smoothly, interrupted periodically by recurrent bursts of activity occurring roughly every 18-24 months. The rates of decline for cycles 21 and 11, some 8 yr into the cycles, are such that had one predicted cycle duration for these cycles, one probably would have called cycle 11 a short-period cycle of duration about 110-120 months, and suspected cycle 21 to behave in similar fashion. However, cycle 11 exhibited a rather long tail of sustained, low, smoothed sunspot number, below approximately 13, which began about 5 yr following R_M and lasted for about 3 yr, thus making cycle 11 a long-period cycle. If cycle 21 is, indeed, a long-period cycle, then, it must soon begin (if it has not already) a long tail of sustained, low, smoothed sunspot number. The smoothed sunspot number for December 1984, 5 yr following R_M, had a value of about 27, 1.8 times the corresponding value for cycle 11.

Using cycle 11 as an indicator of future activity for cycle 21, one anticipates that cycle 21 soon will be leveling-off in smoothed sunspot number, to a value below 20, and will continue a sustained, slow decline for an interval of, perhaps, 3 yr, prior to cycle 22 minimum. An approximately 18-24 month recurrence rate for enhanced activity, which tends to prolong the cycle and delay next cycle minimum, suggests that another "bump" in activity may have occurred or will occur in either late 1985 or early 1986. Cycle 22 minimum is predicted to occur in the range of late 1987 to early 1988.

3. Conclusion

Bimodality of the solar cycle has previously been suggested by Wilson (1984), Sargent (1984b), and Rabin, Wilson, and Moore (1986). In this investigation, bimodality of the solar cycle has been demonstrated using a scatter plot of DES versus ASC. For the well-observed cycles 8-20, long-period cycles almost always lie above the mean descent duration <DES>, while short-period cycles lie below <DES>. Without the invokement of bimodality, DES is not strongly correlated with ASC; however, with its invokement, relatively strong inverse relations ($r^2 > 0.66$) between DES and ASC become apparent, with the inverse relations being significant at the 5% level.

The "period-growth dichotomy" reported in Rabin, Wilson, and Moore (1986) strongly suggests that cycle 21 will be a long-period cycle of duration > 133 months, therefore ending after July 1987. Application of the observed value of ASC for cycle 21 in the linear fit for long-period cycles leads to the prediction that cycle 21 will have a DES near 99 months, with a cycle duration enduring about 141 months. Thus, cycle 21 may not end until late 1987 or early 1988. If, indeed, cycle 21 is a long-period cycle, then it must be considered an anomalous cycle, based on its "first difference" in amplitude, a classification also given to cycles 11 and 13 which like cycle 21 occur on the downward envelope of the sunspot number curve, yet have upward "first differences."

A comparison of the smoothed sunspot number curves for cycles 21 and 11 reveals striking similarity. Both rose to higher-than-average amplitude, single, well-defined peaks (in terms of R_M) in shorter-than-average ascent duration, and both have shown recurrent activity during cycle decline, which tends to prolong the cycle. The enhanced sunspot activity has recurred about every 18-24 months. The remarkable similarity between cycles 21 and 11 suggests that, if cycle 11 can be used as an indicator for future activity in cycle 21, cycle 21 may be entering an interval of sustained, low, smoothed sunspot number, interrupted by brief episodes of slightly enhanced sunspot activity.

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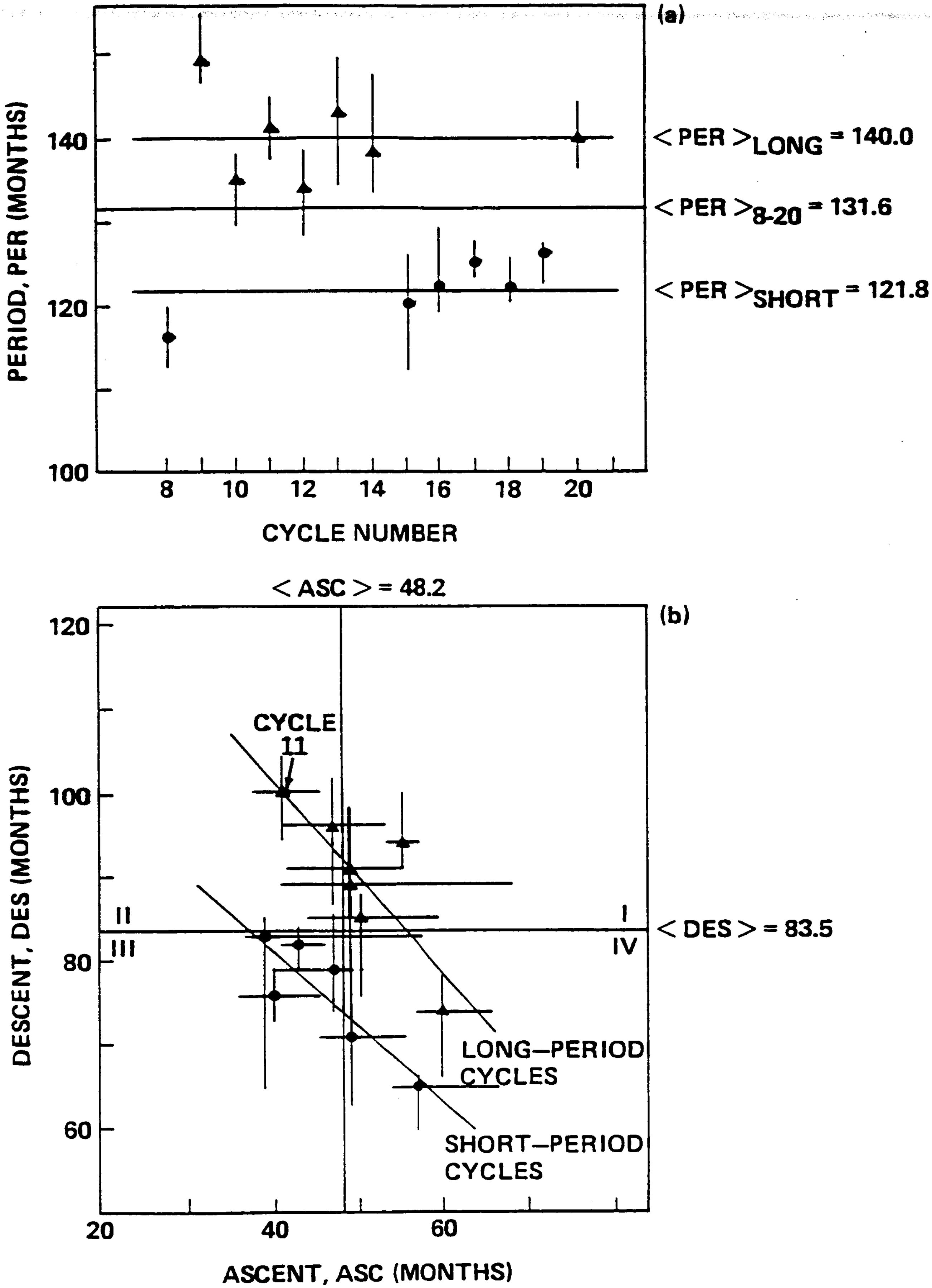
FIGURE CAPTIONS

Figure 1. (a) Cycle duration or period PER (in months) versus cycle number, spanning the well-observed solar cycles. Mean values for cycles 8-20 and for long-period and short-period cycle groupings are identified. Notice that no cycles have been observed to occur in the range 128-133 months, which contains the mean of the entire distribution. Instead, cycle durations appear to cluster near 140 and 122 months, perhaps, suggesting a bimodal classification of solar cycles.
(b) Durations of descent DES versus ascent ASC (in months). Mean values of ASC and DES are identified. Long-period cycles tend to populate quadrants I and II, while short-period cycles populate quadrants III and IV. Linear fits for the two cycle duration classes are shown. Equations for the fits are included in the text. Error bars have been estimated for ASC, DES, and PER.

Figure 2. Smoothed sunspot number for cycles 21 (dots) and 11 (line) versus time (in months) from cycle minimum occurrence. Notice the striking similarity between the two cycles.



(a)
PERIOD, PER (MONTHS)
140
120
100
8
10
12
14
16
18
20
CYCLE NUMBER
< PER >LONG = 140.0
< PER >8-20 = 131.6
< PER >SHORT = 121.8
< ASC > = 48.2
(b)
DESCENT, DES (MONTHS)
120
100
80
60
20
40
60
ASCENT, ASC (MONTHS)
CYCLE 11
I
II
III
IV
< DES > = 83.5
LONG–PERIOD CYCLES
SHORT–PERIOD CYCLES

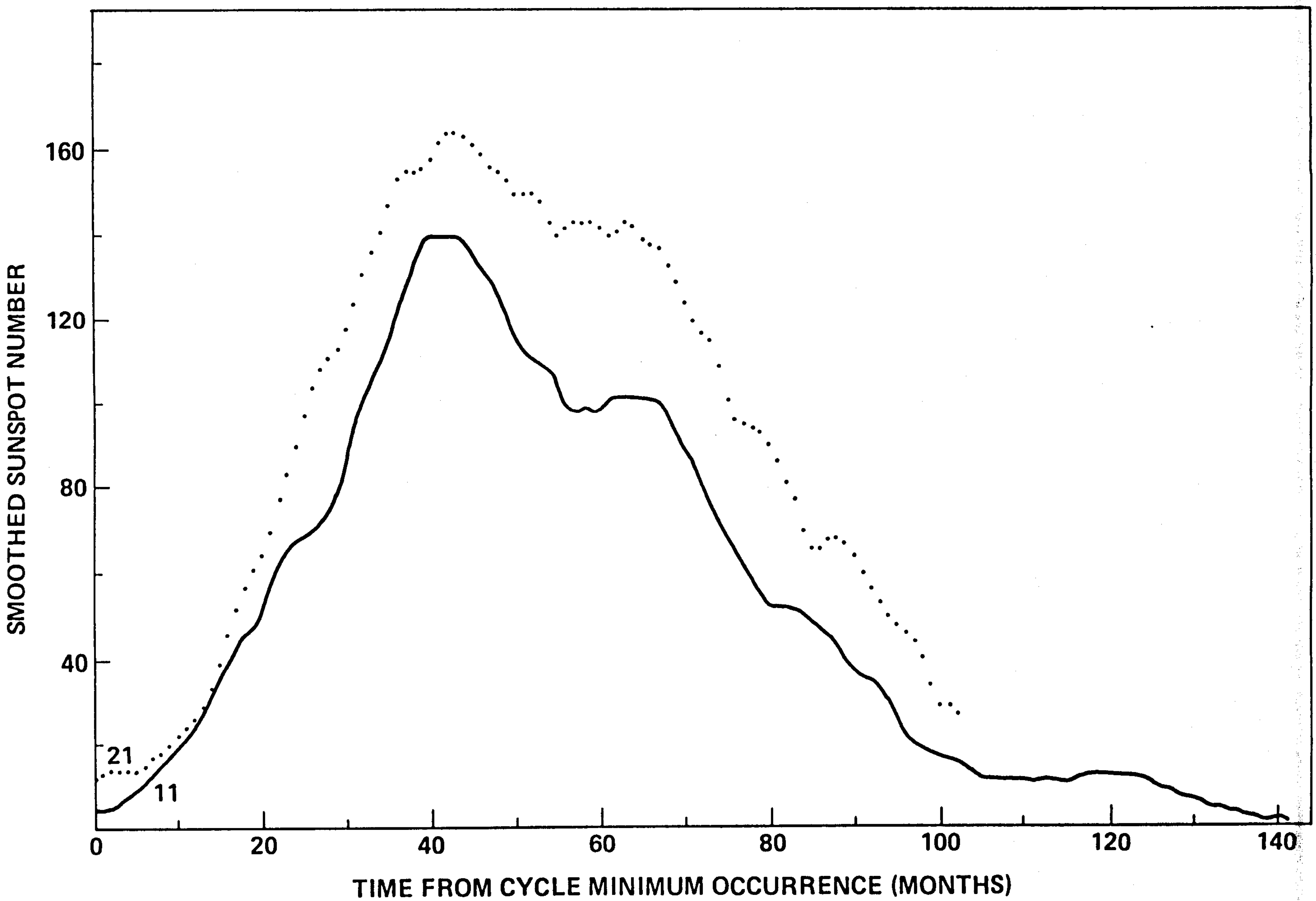

SMOOTHED SUNSPOT NUMBER
160
120
80
40
21
11
0
20
40
60
80
100
120
140
TIME FROM CYCLE MINIMUM OCCURRENCE (MONTHS)

TABLE CAPTIONS

Table 1. Mean values, standard deviations S.D., variances VAR., and 95% and 90% confidence intervals C.I. of the means for cycles 1-7, 8-20, and their respective ratios.

Table 2. Values of ascent ASC, descent DES, and cycle duration or period PER for the well-observed solar cycles. Also identified are the means, standard deviations S.D., 95% and 90% confidence intervals C.I. of the means, and the cycle duration mode, being either long L or short S (in comparison to <PER>).

Table 3. Means, standard deviations S.D., and 95% and 90% confidence intervals C.I. of the means for the two presumed cycle duration modes, long L and short S.

	CYCLES 1 - 7 (n = 7)				CYCLES 8 - 20 (n = 13)				RATIOS = x_{1-7}/x_{8-20}			
	R_M	ASC	DES	PER	R_M	ASC	DES	PER	R_M	ASC	DES	PER
MEAN	95.9	60.4	74.6	135.0	116.2	48.2	83.5	131.6	0.83	1.25	0.89	1.03
S.D.	43.7	20.8	23.8	20.7	38.1	6.5	10.4	10.4	1.15	3.20	2.29	1.99
VAR.	1908.3	431.0	565.6	428.0	1454.9	41.8	107.9	107.3	1.31	10.31	5.24	3.99
95% C.I.	40.4	19.2	22.0	19.1	23.0	3.9	6.3	6.3	1.76	4.92	3.49	3.03
90% C.I.	32.1	15.2	17.5	15.2	18.8	3.2	5.1	5.1	1.71	4.75	3.43	2.98

CYCLE	ASC	DES	PER	MODE
8	40	76	116	S
9	55	94	149	L
10	50	85	135	L
11	41	100	141	L
12	60	74	134	L
13	47	96	143	L
14	49	89	138	L
15	49	71	120	S
16	57	65	122	S
17	43	82	125	S
18	39	83	122	S
19	47	79	126	S
20	49	91	140	L
MEAN	48.2	83.5	131.6	
S.D.	6.5	10.4	10.4	
95% C.I.	3.9	6.3	6.3	
90% C.I.	3.2	5.1	5.1	

MODE	ASC	DES	PER	
S	45.8	76.0	121.8	MEAN
	6.7	6.9	3.6	S.D.
	7.0	7.3	3.8	95% C.I.
	5.5	5.7	3.0	90% C.I.
L	50.1	89.9	140.0	MEAN
	6.0	7.5	5.1	S.D.
	5.6	7.9	4.7	95% C.I.
	4.4	6.2	3.7	90% C.I.